SQ11

We only consider normal Zeeman effect.

$\vec{S} = 0 \Rightarrow \vec{\mu} = \vec{\mu}_L = -\frac{e}{2m_e}\vec{L}$

$$\hat{H}_{Zeeman} = (-\vec{\mu}_L \cdot \vec{B}_{ext}) = \frac{e}{2m_e}\vec{L}\cdot\vec{B}_{ext} = \frac{e}{2m_e}\hat{L}_z B_{ext}$$

(without loss of generality, we assume B field is in z direction)

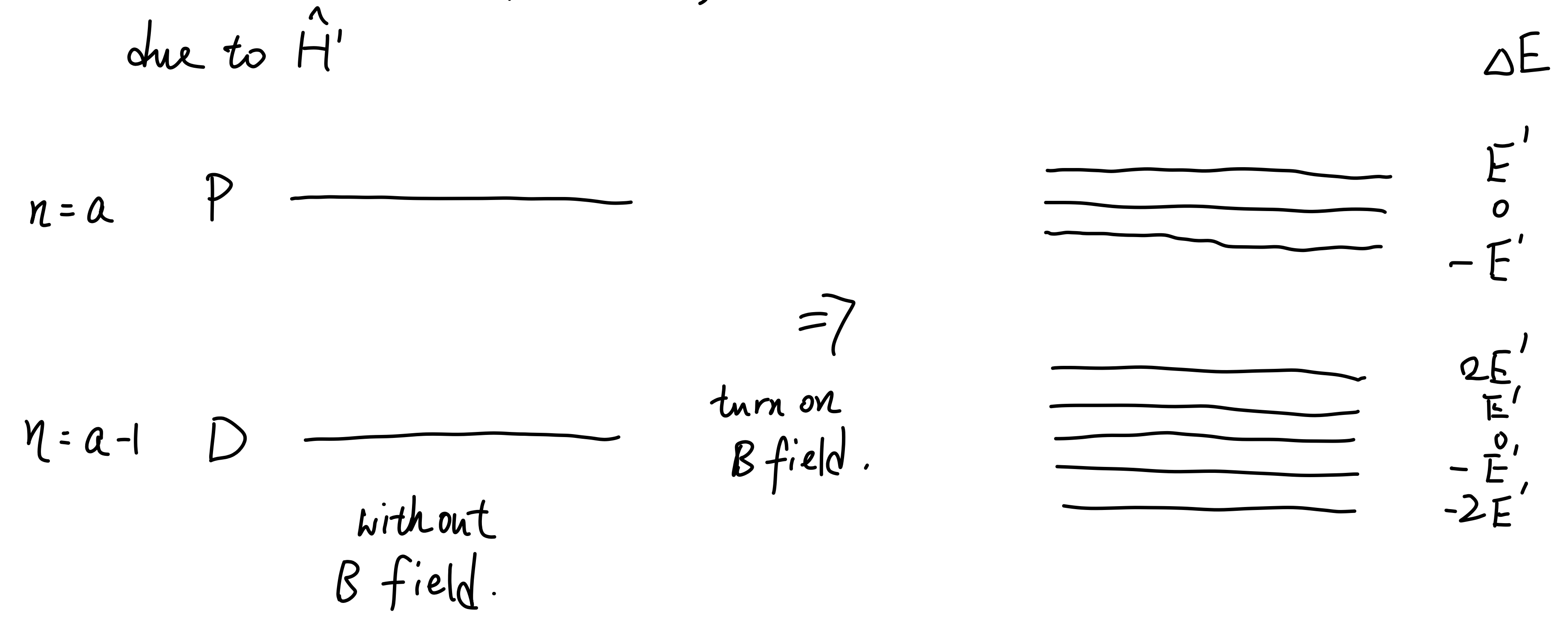
$$E^{(1)}_{Zeeman} = \frac{e\hbar}{2m_e}B_{ext}m_l = E' m_l .$$

Now consider an upper P level and lower D level.

P level : $l=1 \Rightarrow m_l = 1, 0, -1$

D level : $l=2 \Rightarrow m_l = 2, 1, 0, -1, -2$

Under an external B field, The two levels is split into multiple levels due to $\hat{H}'$



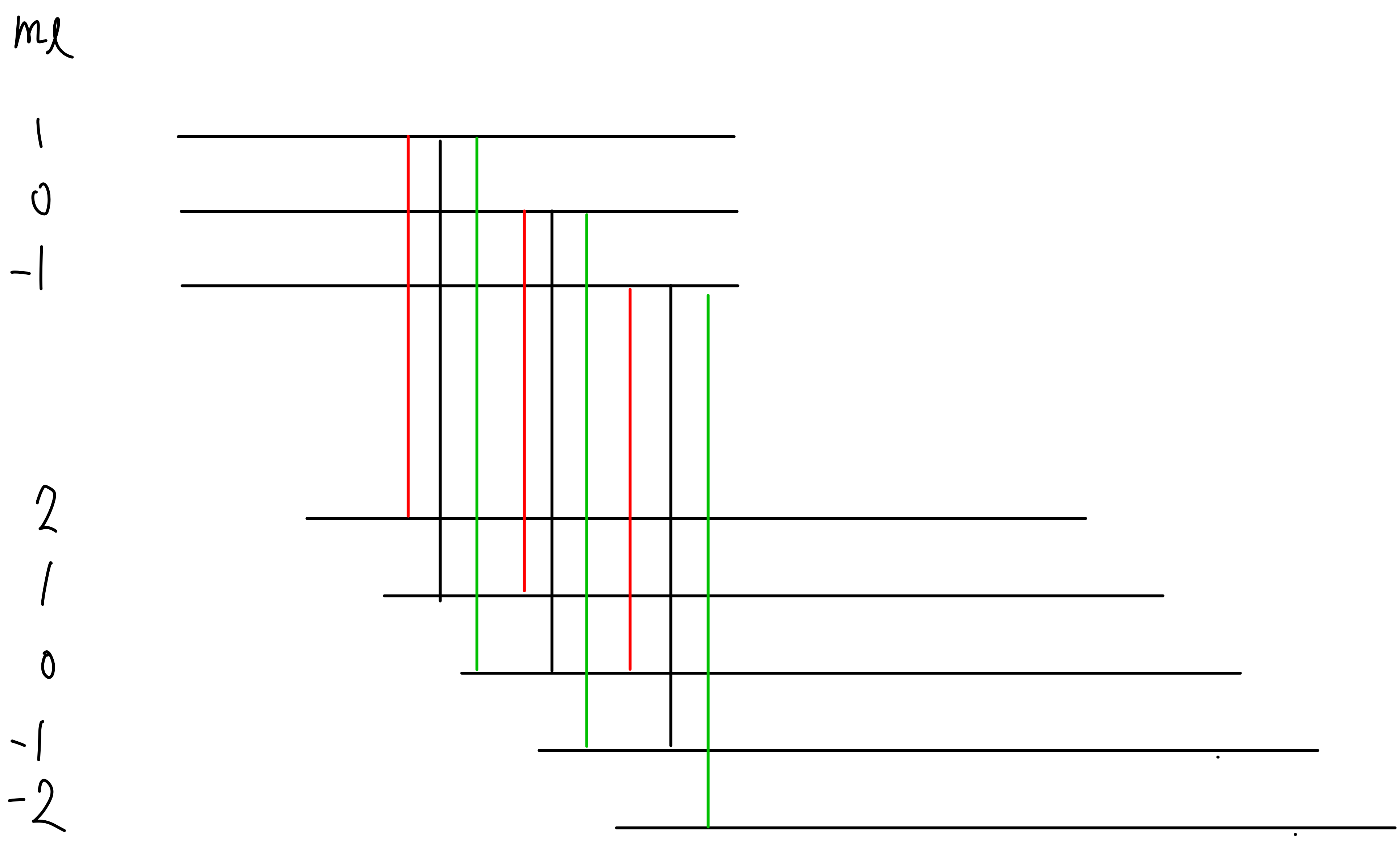
The energy difference of two levels :

$$E_P - E_D = E_a - E_{a-1} + E'(m_P - m_D) = \Delta E + E'\Delta m_l .$$

where m_P and m_D are the m_l of P level and D level

Here ΔE is the 'normal' transition energy, depend only on quantum number n. Substitute $\Delta m_l = 0, \pm 1$, we immediately see that there are two spectral line besides the normal line which corresponds to ΔE.

Supplementary graph



$$E_P - E_D = \Delta E + \begin{cases} +E' \\ 0 \\ -E' \end{cases}$$

In fact it also works for jumping up from D to P

SQ12

Consider Na atom as an effective H atom.

We know the B_{int} would cause a splitting : E_3 —— → $\overset{\textstyle —}{—}\updownarrow 2\mu_B B_{int}.$

From lab, we know the energy difference from the two spectral lines.

$$\Delta E = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = 2\mu_B B_{int}$$

$$\frac{hc}{2\mu_B}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = B_{int}$$

$$\frac{1239.841\ nm}{2 \cdot 5.78838 \cdot 10^{-5} eV\,T^{-1}}\left(\frac{1}{588.995\,nm} - \frac{1}{589.592\,nm}\right) = B_{int}$$

$B_{int} = 18.1034$. Tesla

SQ13

a) $a_0 = 5.29 \times 10^{-11}\, m$

$m_e = 9.109 \times 10^{-31}\, kg$.

$e = 1.602 \times 10^{-19}\, C$

$E_h = 4.360 \times 10^{-18}\, J$

$1\, \hbar$ of $L = 1.055 \times 10^{-34}\, Js$

$= 1.055 \times 10^{-34}\, kg\, m^2 s^{-1}$

$4\pi\varepsilon_0 = 1.113 \times 10^{-10}\, F \cdot m$

b) $E_h = 27.2\, eV$

$= 2 \times 13.6 eV$ related to ground state energy of hydrogen

c) i E_h (eV per atom)

$1\, eV = 1.602 \times 10^{-19}\, J$

$= 1.602 \times 10^{-22}\, kJ$

$1\, mol = N_A$ atoms

1 atom $= \frac{1}{N_A}\, mol$:

$\therefore E_h$ (eV per atom)

$= 27.2 \cdot 1.602 \times 10^{-22} \cdot \frac{1}{\frac{1}{6.022 \times 10^{23}}} (kJ\, mol^{-1})$

$= 27.2 \times 1.602 \cdot 60.22\ (kJ\, mol^{-1})$

$= 2624\ (kJ\, mol^{-1})$

ii From i), $1 eV = 1.602 \cdot 60.22 = 96.5 kJ/mol \approx 100 kJ/mol$ worth remembering.

$5.1 eV = 5.1 \cdot 96.5 = 492 (kJ\, mol^{-1})$

d) $\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0 r}$

Since $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

make x, y, z dimensionless $\Rightarrow$ $x' = \frac{x}{a_0}$ $\quad r' = \sqrt{x'^2 + y'^2 + z'^2}$

$y' = \frac{y}{a_0}$ $\quad = \frac{r}{a_0}$

$z' = \frac{z}{a_0}$

New $\nabla'^2 = \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \frac{\partial^2}{\partial z'^2}$

$= a_0^2 \nabla^2$

$-\frac{\hbar^2}{2ma_0^2}\nabla'^2$

Recell bohr radius $a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2}$

$\frac{\hbar^2}{ma_0^2} = \frac{\hbar^2}{m_e a_0} \frac{m_e e^2}{4\pi\varepsilon_0 \hbar^2}$

$= \frac{e^2}{4\pi\varepsilon_0 a_0} = E_h$

$\therefore \hat{H} = -\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0 r}$

$= -\frac{E_h}{2}\nabla'^2 - \frac{E_h}{r'}$

$\frac{\hat{H}}{E_h} = \hat{H}$ (atomic units) $= -\frac{1}{2}\nabla'^2 - \frac{1}{r'}$ which is now dimensionless